Missile Deployment Module Accuracy Verification

Michael L. Howard*
Tracor Aerospace, Austin, Texas

As part of an overall missile error budget, an allocation is made to the deployment module of allowable delta velocity and tipoff rate imparted to a re-entry vehicle at release. This paper presents a method of verifying compliance with these accuracy requirements using a limited amount of flight test data. A brief desription is given of a point estimation scheme that can be employed to obtain estimates of single samples of delta velocity and tipoff rate from flight test data. The remainder of the paper then concentrates on presenting a unique hypothesis testing method, based on a fusion of established statistical methods, which is specifically designed to address the stated requirements verification problem.

Nomenclature

f	= density of mean zero, variance 1 normal random variable
GLB(H)	= greatest lower bound of number set H
$h \epsilon H$	= h is a member of set H
M^T	= transpose of matrix M
$P(A)$	= probability of occurence of event A
t_q	= density of student's t distribution with q degrees of freedom
V^-	= estimate of re-entry vehicle inertial velocity just prior to release
V^+	= estimate of re-entry vehicle inertial velocity just after release
ΔV	= re-entry vehicle inertial velocity change due to release
ΔV^-	= initial estimate of ΔV
ΔV^+	= final estimate of ΔV
$\|X\|$	= Euclidian norm of vector X
$\bar{x}$	= mean of random variable x
$\|x\|$	= absolute value of variable x
α	= test level
β	= test power
ρ	= correlation coefficient
σ	= standard deviation of random variable x
χ_q^2	= density of Chi-squared distribution with q degrees of freedom
ω	= re-entry vehicle inertial lateral rate just after release
$\hat{\omega}$	= estimate of ω

I. Introduction

A PORTION of a missile system error budget is often allocated to the re-entry vehicle (RV) deployment module (DM) for RV release. For the MX missile, this allocation consists of a maximum 99 percentile lateral tipoff rate of 0.7 deg/s and a maximum CEP, CEP_{DM}, induced by the RV inertial velocity change due to DM/RV separation. For both of these requreiments, it is assumed that a prescribed trajectory will be flown and that specified limit cycle conditions will exist at RV release.

The verification of these DM accuracy requirements is a formidable problem. The tightness of the associated design requirements does not allow for comfortable design margins. In addition, fewer than 27 inertially instrumented RVs will be released throughout the test program, thus limiting the number of observations of release. Earlier missile development programs have not faced a deployment accuracy verification problem of this difficulty.

A unique approach to this problem based on a fusion of known statistical methodology is presented here. A brief discussion of the data reduction scheme to be employed per RV release is given first. Then a common mathematical framework for the statement of both requirements is developed. A discussion of the use of a priori models and of modeling assumptions then follows. The remainder of the paper is devoted to developing and illustrating hypothesis testing techniques designed to verify compliance of performance with the CEP requreiment. Similar but unique techniques can be applied to the tipoff rate performance evaluation.

II. Data Reduction per RV Release

Three data sources will be utilized for the MX missile in reconstructing the kinematics of an RV immediately prior to and following its release: 1) linear and angular position and velocity from the missile inertial instruments, 2) linear and angular velocity form the RV inertial instruments, and 3) relative velocity from a linear velocity transducer attached to the DM at the RV base. Since the release time is short (≤ 5 ms) the release force will be modeled as an impulse. The minute level of the velocity to be estimated precludes the use of data taken during periods of active attitude control. Hence less than 1 s of data must be uded to reconstruct $\Delta V = [\Delta V_x \Delta V_y \Delta V_z]^T$ and $\omega = [\omega_y \omega_z]^T$. Data types 1 and 2 will be processed in a six degree of freedom Kalman filter for the data time interval prior to release to obtain an estimate V^- of RV velocity prior to release. All data types will be processed in a six degree-fo-freedom Kalman smoother for the data time interval after release to obtain estimates V^+, ΔV^-, and $\hat{\omega}$, of the RV velocity after release and of ΔV and ω. Then V^-, ΔV^-, and V^+ will be optimally combined to produce a final estimate ΔV^+ of ΔV. These point estimates of ΔV and ω, together with their associated error covariances, are single observation inputs to the hypothesis testing performance verification process.

While the above data reduction scheme clearly requires a substantial effort by a estimator theory specialist, it is based on well-known methods.[2,3] The hypothesis testing technique to which the remainder of this paper is devoted is, in contrast, a unique method presented here for the first time.

III. Accuracy Requirements Framework

The two DM requirements both conform to the following framework: X is a two-dimensional random vector, a and b

Presetned as Paper 81-2364 at the AIAA/SETP/SFTE/SAE/ITEA/IEEE 1st Flight Testing Conference, Las Vegas, Nev., Nov. 11-13, 1981; submitted Dec. 9, 1981; revision received April 1, 1983.

*Engineering Manager. Systems Analysis, Countermeasures Engineering. Member AIAA.

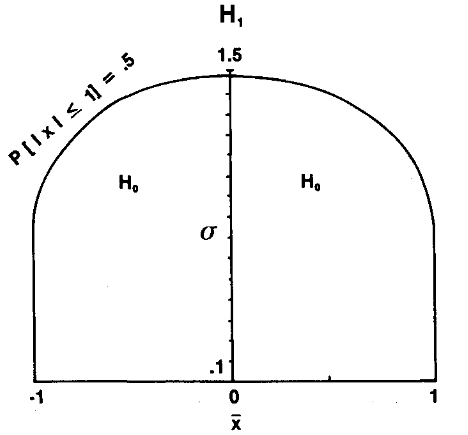

Fig. 1 Graph of H_0 (under curve) and H_1 (over curve).

are positive numbers with $b<1$, and it is required that

$$P(|X| \leq a) \geq b \tag{1}$$

be satisfied. For the tipoff rate requirement, $X=\omega$, $a=0.7$ deg/s, and $b=0.99$. For the CEP requirement, $X=M\Delta V$, where M is the appropriate 2×3 miss partials matrix, $a=\mathrm{CEP}_{\mathrm{DM}}$, and $b=0.5$. Given similar modeling assumptions (see Sec. IV), the testing techniques presented here will apply equally well to any requirement of Eq. (1).

IV. A Priori Models and Modeling Assumptions

The basic concept of hypothesis testing is to use test data to decide between a primary or null hypothesis and one or more alternate hypotheses concerning the nature of the phenomenon being observed. For the current application, the null hypothesis will be a statistical description of X produced by Monte Carlo simulation of RV release dynamics and, in the case of ΔV, application of the miss partials matrix. It is assumed that X is normally distributed. Design validation prior to flight test consists of showing that the null hypothesis produced by simulation inducates that Eq. (1) will be satisfied.

The instrumented RVs to be released will be distributed among the 12 RV stations on the DM. Simulation results indicate that the ΔV statistics will vary very little from station to station, but notable differences occur in the ω statistics. Further discussion of the ω requirements verification method is deferred. The remainder of this presentation will concentrate on the CEP requirement. In particular, X is assumed to be $M\Delta V$, as described in Sec. III.

The assumption is made that ΔV is not dependent on RV station. This means each instrumented RV successfully released provides an independent observation of ΔV. The miss partials matrix M, however, is RV station dependent. Hence, there are actually 12 different models to test, each using the same set of ΔV observations. It is also assumed that the mean ΔV will be removed by targeting on the operational system. Thus, $X=M\Delta V$ is mean zero if the null hypothesis is correct. Note, however, that X need not be mean zero if some alternate hypothesis is correct.

V. Simplified Test Formulation

The detailed test formulation is presented in Sec. VI. A simplified formulation is given here to illustrate the essential details of the actual test. Consider a normally distributed random variable x. In analog with Eq. (1), the requirement to be verified is taken to be

$$P(|x| \leq 1) \geq 0.5 \tag{2}$$

where the value 1 is chosen arbitrarily. The distribution of x is characterized by the vector $h=[\bar{x},\sigma]^T$. The null hypothesis will thus be represented by a member $h^{(0)}$ of the set H_0 consisting of all values of h that allow Eq. (2) to be satisfied. The set H_1 of alternatives will consist of all values of h that do not allow Eq. (2) to be satisfied. The boundary B between H_0 and H_1 consists of all values of h that allow

$$P(|x| \leq 1) = 0.5 \tag{3}$$

to be satisfied. A graph of H_0, B, and H_1 in the $(\bar{x},\sigma)$ plane is shown in Fig. 1. The null hypothesis is

$$h=h^{(0)} \tag{4}$$

and the alternative is

$$h \in H_1 \tag{5}$$

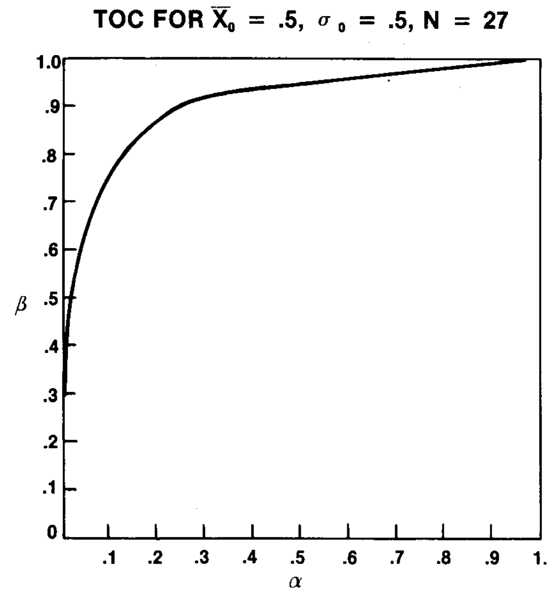

Fig. 2 Test operating characteristic curve.

The hypothesis test will be a joint test based on separate classical tests of the mean and variance. A key probability is the test level, the probability of rejecting the null hyupothesis if it is true. The null hypothesis is accepted at a given level α only if both the subhypotheses $\bar{x}=\bar{x}_0$ and $\sigma=\sigma_0$ are accepted. The level α required for the two subtests, in order that the joint test have a prescribed level α, can be computed numerically. The individual tests of the mean and variance will now be presented. It is assumed that noise-free observations $x_1,\ldots,x_n$ of x have been obtained.

Mean Test

The sample mean is $m=(x_1+\ldots+x_n)/n$ and the sample variance is $s^2=[(x_1-m)^2+\ldots+(x_n-m)^2]/(n-1)$. The test statistic for $\bar{x}$ is $t=\sqrt{n}(m-\bar{x}_0)/s$. For a given level α' for this subtest, $\bar{x}=\bar{x}_0$ is accepted only if $|t|<c$, where c is given by

$$\int_0^c t_{n-1}(r)\,\mathrm{d}r=(1-\alpha')/2$$

The power of a test is the probability of rejecting the null hypothesis for that test when the alternative hypothesis is true. The power of this test against an alternative $h^{(1)}=[\bar{x}_1,\sigma_1]^T$ is

$$\beta'=1-\int_{-a-b}^{a-b} f(s)\,\mathrm{d}s$$

Table 1 27 Random samples

0.59692925	0.11348933	0.73820746
2.88640594	2.27994823	−0.50739443
1.17621040	0.26308960	−0.22926009
0.55161661	1.93351269	−0.18456709
0.83270508	1.34008598	2.08885479
0.31785572	0.51687068	−0.14415038
−0.01646888	1.01307869	1.38148117
0.36783260	1.56136799	2.32853699
0.09254599	0.59382957	

Table 2 Acceptance intervals

α	α	C	C_1	C_2
0.002	0.001	3.707	8.802	57.68
0.021	0.01	2.779	11.48	49.42
0.09	0.05	2.056	14.24	42.93
0.19	0.1	1.706	15.82	39.83
0.36	0.2	1.315	17.77	36.44
0.52	0.3	1.058	19.19	32.26
0.66	0.4	0.856	20.36	32.60
0.97	0.8	0.256	24.20	27.88

where $b=(n-1)(\bar{x}_1-\bar{x}_0)/\sigma_1$, and $a=cs/\sigma_1$ (see REf. 3). The power is a function of the random observations. An upper bound β'_w for β' that is independent of the observations will be used in place of β'. See the Appendix for the derivation of β'_w.

Test of the Variance

The test statistic for σ is $k=(n-1)s^2/\sigma_0^2$. For a given choice of the level α' for this subtest, the hypotehsis $\sigma=\sigma_0$ is accepted only if $c_1<k<c_2$, where c_1 and c_2 are determined by the equations

$$\int_{c_1}^{c_2} \chi^2_{n-1}(r)\,\mathrm{d}r = 1-\alpha' = \int_{c_1}^{c_2} \chi^2_{n+1}(r)\,\mathrm{d}r$$

This corresponds to a two-sided test of variance. In this setting a one-sided test would be more natural, but the two-sided test is employed to maintain the analog with the actual test (see Sec. VI). The power of the test against an alternative is $\sigma=\sigma_1$ is

$$\beta' = 1-\int_{r_1}^{r_2} \chi^2_{n-1}(r)\,\mathrm{d}r$$

where $r_1=c_1\sigma_0^2/\sigma_1^2$ and $r_2=c_2\sigma_0^2/\sigma_1^2$.

Test Operating Characteristic

A plot of the power β vs the level α of the joint test is called the test (or receiver) operating charactreistic (TOC). The computation of a conservative TOC prior to the availability of test data is need in order to estimate how meaningful the test will be and to trade off β and α in choosing an accept/reject decision criterion.

The power β of the test of $\boldsymbol{h}=\boldsymbol{h}^{(0)}$ vs $\boldsymbol{h}\epsilon H_1$ at a given level α is GLB $\{\beta_1:\beta_1$ is the power of the test at level α of $\boldsymbol{h}=\boldsymbol{h}^{(0)}$ vs $\boldsymbol{h}=\boldsymbol{h}^{(i)}$ for some $\boldsymbol{h}^{(1)}\epsilon H_1\}$. It can be shown that β is also GLB $\{\beta_1:\beta_1$ is the power of the test at level α of $\boldsymbol{h}=\boldsymbol{h}^{(0)}$ vs $\boldsymbol{h}=\boldsymbol{h}^{(i)}$ for some $\boldsymbol{h}^{(1)}\epsilon H_1\}$. Use of the last expression simplifies the computational procedures. The power β_1 of the test of $\boldsymbol{h}=\boldsymbol{h}^{(0)}$ vs an alternative $\boldsymbol{h}=\boldsymbol{h}^{(i)}$ at level α can be show to be bounded below by Max (β_m,β_s), where β_m and β_s are the powers of the mean and variance subtests, respectively. Thus, Max (β_m,β_s) may be used to compute a conservative TOC. For a given $\boldsymbol{h}^{(0)}$, a given level α, and a given sample size n, the boundary B is searched and $\beta=$ GLB $\{\mathrm{Max}(\beta_m,\beta_s):\boldsymbol{h}^{(0)}\epsilon B\}$ is obtained. The TOC for $\boldsymbol{h}^{(0)}=[0.5,0.5]^T$ and $n=27$ is presented in Fig. 2.

Test Application Example

The following example will illustrate how the test would be applied. Assume that $\boldsymbol{h}^{(0)}=[0.5,0.5]^T$ and the "truth" $\boldsymbol{h}$ is $[0.9,1.066]^T$. The value of $\boldsymbol{h}$ was chosen on B. Since the "truth" corresponds to marginal performance, the test should reject $\boldsymbol{h}^{(0)}$. A set of 27 observations based on $\boldsymbol{h}$ were simulated (see Table 1). The statistical values computed were $m=0.7119$, $s=1.114$, $t=-0.8774$, and $k=129.1$. The subtest level and acceptance parameters for various levels α of the joint test are given in Table 2. Due to the value of k, $\boldsymbol{h}=\boldsymbol{h}^{(0)}$ will be accepted only at low levels, i.e., $\alpha<0.002$. The TOC (Fig. 2) shows very poor power at these levels, i.e., $\beta<0.05$. Any reasonable decision criterion will therefore result in rejection of $\boldsymbol{h}=\boldsymbol{h}^{(0)}$ and acceptance of the alternative, i.e., Eq. (2) is not met. Notice that the vector $[ms]^T$ lies in H_0 (see Fig. 1). Thus, a naive reliance on the sample mean and variance alone would lead to a decision to accept H_0, i.e., that Eq. (2) is met by x.

VI. Test Formulation

The down/cross-range miss vector $X=[X_{\mathrm{DR}},X_{\mathrm{CR}}]^T$ is statistically characterized by its mean $\bar{X}=[\bar{X}_{\mathrm{DR}},\bar{X}_{\mathrm{CR}}]^T$ and by its covariance matrix

$$C_X=\begin{bmatrix} \sigma^2_{\mathrm{DR}} & \rho\sigma_{\mathrm{DR}}\sigma_{\mathrm{CR}} \\ \rho\sigma_{\mathrm{DR}}\sigma_{\mathrm{CR}} & \sigma^2_{\mathrm{CR}} \end{bmatrix}$$

where $C_X=MC_{\Delta V}M^T$. Hence, a hypothetical distribution for X can be represented by a vector $\boldsymbol{h}=[\bar{X}_{\mathrm{DR}},\bar{X}_{\mathrm{CR}},\sigma_{\mathrm{DR}},\sigma_{\mathrm{CR}},\rho]^T$. This generalizes the situation of the simplified test formulation. Continuing the analogy, H_0 is the set of all choices of $\boldsymbol{h}$ that allow

$$P(|X|\leq \mathrm{CEP_{DM}})\geq 0.5 \tag{6}$$

to be satisfied, H_1 *is the set of all choices of* $\boldsymbol{h}$ that do not allow Eq. (6) to be satisfied, and B is the set of all $\boldsymbol{h}$ that allow

$$P(|X|\leq \mathrm{CEP_{DM}})=0.5 \tag{7}$$

to be satisfied. Since H_0, H_1, and B are subsets of a five-space, graphic representation of these sets is not possible.

The a priori model of X is a vector in H_0. The hypothesis test will be formulated to test

$$\boldsymbol{h}=\boldsymbol{h}^{(0)} \tag{8}$$

against the alternative

$$\boldsymbol{h}\epsilon H_1 \tag{9}$$

It will be a joint test consisting of subtests of $\bar{X}^{(0)}_{\mathrm{DR}}$, $\bar{X}^{(0)}_{\mathrm{CR}}$, and $C_X^{(0)}$, the mean components and covariance matrix corresponding to $\boldsymbol{h}^{(0)}$. The level α of the joint test can be computed numerically given a common level α' for the three subtests. The observations of X will be of the form $\hat{X}^{(i)}=X^{(i)}+E^{(i)}, i=1,\ldots, n$, where $X^{(1)},\ldots,X^{(n)}$ are the actual (noise-free) observations of X and $E^{(i)}$ represents the error in the estimate of $X^{(i)}$. It is assumed that $X^{(i)},\ldots,X^{(n)}$, $E^{(1)},\ldots, E^{(n)}$ are independent and the $E^{(i)}$ are normally distributed and mean zero. The $\hat{X}^{(i)}$ are obtained from observations of $\boldsymbol{\Delta V}$. The individual tests of the mean components and of the covariance will now be presented. Then a brief description of how the TOC curve for the joint test can be computed will be given.

Test of the Mean Components

The mean test in Sec. V will be used for testing $\bar{X}_{DR}^{(0)}$ and $\bar{X}_{CR}^{(0)}$ individually. The test for $\bar{X}_{DR}^{(0)}$ will be developed; the test for $\bar{X}_{CR}^{(0)}$ is similar. Let $x = X_{DR}$ and $e = E_{DR}$. Thus, the assumption is made that $E_{DR}^{(1)},\ldots, E_{DR}^{(n)}$ have the same variance, so that they may be regarded as observations from a single random variable E_{DR}. The variances of x and e are $\sigma^2 = C_X(1,1)$ and σ_e^2. Let $e_i = E_{DR}^{(i)}$, $i = 1,\ldots, n$. The observations to be employed in the mean test are $\hat{x}_i = x_i + e_i$, $i,\ldots,n$. It follows from the assumptions made that $\hat{x}_1,\ldots,\hat{x}_n$ are independent and that each is normally distributed with mean $\bar{x}$ (the mean of x) and variance $\hat{\sigma}^2 = \sigma^2 + \sigma_e^2$. The test is formulated as in Sec. V, using $\hat{x}$ in place of x. For a given level α' for this subtest and for a given alternative $h^{(1)}$, a worst case power of the test that is independent of the observation values can be computed (see Appendix).

Test of the Covariance

Let $P^{(1)},\ldots, P^{(n)}$ denote the covariances of $E^{(1)},\ldots, E^{(n)}$, respectively, and let $C^{(i)} = C_X^{(0)} + P^{(i)}$, $i = 1,\ldots, n$. Also let $m = (\hat{X}^{(1)} + \ldots + \hat{X}^{(n)})/n$. The test statistic is the following generalization of test statistic k presented in Sec. VI:

$$\hat{K} = \sum_{i=1}^{n} (\hat{X}^{(i)} - m)^T (C^{(i)})^{-1} (\hat{X}^{(i)} - m)$$

If $C_X = C_X^{(0)}$ then $\hat{K}$ is a χ^2 variable with $2n-2$ degrees of freedom. For a given level α' for this subtest, $C_X = C_X^{(0)}$ is accepted only if $c_1 < \hat{K} < c_2$, with c_1 and c_2 given by

$$\int_{c_1}^{c_2} \chi^2_{2n-2}(r)\,\mathrm{d}r = 1 - \alpha' = \int_{c_1}^{c_2} \chi^2_{2n}(r)\,\mathrm{d}r$$

This formulation is a two-sided test of $C_X^{(0)}$. If C_X is not $C_X^{(0)}$, then the distribution of K is not easily characterized. For a given level α' and a given alternative $h^{(1)}$, the power of the test can be computed using the Monte Carlo method.

TOC Computation

The power of the test of $h = h^{(0)}$ against the alternative $h = h^{(1)}$, at a level α, is bounded below by $\beta_1 = \text{Max}(\beta_{m1}, \beta_{m2}, \beta_C)$ whee β_{m1}, β_{m2}, and β_C are the powers of the tests of $\bar{X}_{DR}$, $\bar{X}_{CR}$, and $C_X^{(0)}$, respectively, against $h = h^{(1)}$, each at the appropriate sublevel α'. This is directly analogous to the simplified test formulation. Continuing the analogy, for a given level α a lower bound for the power β of the test of $h = h^{(0)}$ against the alternative $h \epsilon H_1$ is $\beta_L = \text{GLB}\{\beta_1 : h^{(1)} \epsilon B\}$. Hence, a prerequisite for the computation of the conservative TOC is the computation of B. Then, for each level α, β_1 must be evaluated on B and β_1 searched for. A considerable computational burden can be alleviated by proper structuring of the required software. Further discussion along this line is beyond the scope of this presentation.

VII. Summary

An approach to using flight test data in verifying the reentry vehicle release accuracy requirements of a missile deployment module has been outlined. The details of the testing method for the CEP requirement were presented, but presentation of similar details for the lateral tipoff rate requirement was deferred.

The technique developed for the CEP requirement verification may be applicable to verifying the system CEP requirement for a weapon system. In order to accomplish this, it will be necessary to analytially correct all observed misses to a common nominal trajectory or to employ simulated miss vector observations from the entire operational trajecory envelope in computing the a priori model.

Appendix

The nomenclature and situation presented under the heading 'mean test' in Sec. V will be assumed here. The variable s can be rewritten as $s = \sigma_1 k^{1/2}/(n-1)$ where k is a χ^2_{n-1} random variable. Hence, $a = ck^{1/2}/(n-1)$. If $r \geq 0$, then $P(a > r) = P[k > (n-1)(r/c)^2]$. The last expression allows the equation $P(a > r) = 0.01$ to be solved for r, to provide a 99 percentile value for a. A worst case value for the power of the test at level α' is then

$$\beta' = 1 - \int_{-r-b}^{r-b} f(u)\,\mathrm{d}u$$

Acknowledgments

The author wishes to thank Dr. R.G. Bellaire and R.C. Munsterteiger for their support at Avco Systems Division, where this paper's concepts were developed. Special thanks are due to Dr. W.E. Vander Velde of MIT for improving the author's basic knowledge of statistics.

References

[1] Gelb, Arthur ed., *Applied Optimal Estimation,* MIT Press, Cambridge, Mass., 1975.

[2] Bryson, Arthur Jr. and Ho, Yu-Chiu, *Applied Optimal Control,* John Wiley & Sons, New York, 1975.

[3] Lehmann, E.L., *Testing Statistical Hypothesis,* John Wiley & Sons, New York, 1959, p. 199.